Attraction between pancakes vortices in the crossing lattices of layered superconductors

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The intervortex interaction is investigated in very anisotropic layered superconductors in tilted magnetic field. In such a case, the crossing lattice of Abrikosov vortices (AVs) and Josephson vortices (JVs) appears. The interaction between pancakes vortices (PVs), forming the AVs, and JVs produces the deformation of the AV line. It is demonstrated that in the result of this deformation a long range attraction between AVs is induced. This phenomenon is responsible for the dense vortex chains formation. The vortex structure in weak perpendicular magnetic field is the vortex chain phase, while in higher field a more complicated mixed vortex chain-vortex lattice phase emerges.

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Vortex physics in layered superconductors occurred to be extremely rich and interesting. In moderately anisotropic superconductors, a tilted magnetic field leads to the formation of vortices inclined towards the superconducting layers. The interaction between such tilted vortices happens to be quite unusual.

In the plane defined by the vortex line direction and the **c**-axis (normal to the superconducting planes), the interaction between tilted vortices is attractive at long distances. Such attractive intervortex interaction is quite unexpected and leads to the formation of vortex chains, where the intervortex distance is governed by the balance between the long range attraction and the short range repulsion. In layered superconductors, the existence of these vortex chains in tilted field has been predicted in [1] and [2], and subsequently confirmed by the decoration technique [3], and the scanning-tunneling microscopy [4] measurements, in $YBa_2Cu_3O_7$ and $NbSe_2$ crystals respectively. The systems studied in [3,4] are characterized by a moderate anisotropy. The theoretical approach [1,2] is completely applicable to this case and gives a good qualitative and quantitative description of this phenomenon. On the other hand, in the much more anisotropic $Bi_2Sr_2CaCu_2O_x$ (BSCCO) single crystals [5,6], a more complicated mixed vortex chain-vortex lattice phase has been observed. As it has been demonstrated [7], in strongly anisotropic layered superconductors, in a tilted magnetic field, a crossing lattice of Abrikosov and Josephson vortices (JVs), a "combined lattice", must exist. The AV is in fact a line of pancake vortices (PVs) [8–11] interacting with JVs. Following [12], the perpendicular vortex line formed by the PVs is deformed and attracted by JVs, so the JVs stacks accumulate additional PVs, creating vortex row with enhanced density [5,6]. This scenario has been proposed in [12] to explain the mixed chain-lattice state formation. Very recently, using scanning Hall probe microscopy, the detailed studies of vortex chains in BSCCO have been performed and revealed the stability of the dense vortex chains state even in the absence of lattice and in very weak perpendicular magnetic field [13].

In the present work, we demonstrate that the formation of such dense vortex chain is due to the attraction between the deformed lines of pancakes vortices. The deformation, responsible for a long range attraction, appears due to the interaction with JVs. In the result, the mechanism of vortex chains formation, in tilted field, in strongly anisotropic superconductors, appears to be quite similar to the case of moderately anisotropic superconductors [1,2].

Further on, keeping in mind BSCCO, we will consider layered superconductors with high anisotropy ratio $\gamma = \lambda_c/\lambda_{ab} \sim 200-500$, where λ_c is the penetration depth for currents along **c**-axis (perpendicular to the layers), and λ_{ab} is the penetration depth for currents in the ab plane (parallel to the layers). The in-plane field $B_x = B\cos\theta$ penetrates inside the superconductor in the form of JVs, while the perpendicular field $B_z = B\sin\theta$ creates the PVs which interact with JVs via the Josephson coupling [12,14]. We consider the case of a very weak coupling of the layers when the Josephson's core radius $\lambda_J = \gamma s$ (s is the interlayer spacing), is larger than an in-plane penetration depth, i.e. $\lambda_J > \lambda_{ab}$. In [14], the limit of high field oriented near the $(\mathbf{a}, \mathbf{b})$ plane $B \gg H_0 = \phi_0/\gamma s^2$ was considered and it was demonstrated that a zigzag displacement of PVs along **x**-axis is produced, see Fig. 1. The authors of [14] analyzed the case of the dense vortex lattice formed by PVs, but their method to treat the JVs and PV interaction permits us to calculate the shape of the single zigzag line of PVs too.

In the limit of weak Josephson coupling, the interaction which stabilizes the straight PVs line is mainly of electromagnetic origin. Then, we may use the general expression for the energy of an arbitrary configuration of pancakes in the framework of the electromagnetic model [8] to calculate the energy increase due to the line deformation

$$E_{em} = \frac{s}{8\pi\lambda_{ab}^2}\sum_n \int \frac{d^2\mathbf{k}}{(2\pi)^2}\left|\mathbf{\Phi}_n(\mathbf{k})\right|^2 - \mathbf{\Phi}_n(-\mathbf{k}) \times \sum_m \mathbf{\Phi}_m(\mathbf{k})\frac{\sinh(kd)}{2\lambda_{eff}k}\frac{(G_k - \sqrt{G_k^2-1})^{|n-m|}}{\sqrt{G_k^2-1}}, \quad (1)$$

where $\lambda_{eff} = \frac{\lambda_{ab}^2}{s}$, $\mathbf{\Phi}_n(\mathbf{k})$ is the Fourier transform of the total London vector of the n^{th} layer $\mathbf{\Phi}_n(\overline{\mathbf{r}}) = \sum \mathbf{\Phi}(\overline{\mathbf{r}} -$

$\mathbf{r}_{pancake,n}$) (the sum is over all the pancakes of the n^{th} layer), $\mathbf{\Phi}(\mathbf{k}) = i\phi_0\frac{(\mathbf{k}\times\mathbf{z})}{k^2}$, the function $G_k = \cosh(ks) + \frac{\sinh(ks)}{2\lambda_{eff}k}$. For the zigzag deformation with an amplitude u, we have $\mathbf{\Phi}_{2n}(\mathbf{k}) = e^{i\mathbf{uk}}\mathbf{\Phi}(\mathbf{k})$ and $\mathbf{\Phi}_{2n+1}(\mathbf{k}) = e^{-i\mathbf{uk}}\mathbf{\Phi}(\mathbf{k})$, and the energy increase per one layer occurs to be

$$\delta E_{em} = s\left(\frac{\phi_0}{4\pi\lambda_{ab}}\right)^2\frac{u^2}{\lambda_{ab}^2}\ln\left(\frac{\lambda_{ab}}{u}\right). \tag{2}$$

On the other hand, the gain of the Josephson energy due to PVs displacements, calculated following [14] is $\delta E_J = -u\left(\frac{\phi_0}{4\pi\lambda_{ab}}\right)^2\frac{2\phi_0}{\pi\gamma^2 s^2 B_x}$. Minimization of the total deformation energy $\delta E_{em} + \delta E_J$ with respect to u gives its equilibrium value

$$u \sim \frac{\phi_0\lambda_{ab}^2}{\gamma^2 s^3 B_x}\frac{1}{ln\left(\frac{\gamma^2 s^3 B_x}{\phi_0\lambda_{ab}}\right)} = \frac{H_0}{B_x}\frac{\lambda_{ab}^2}{\lambda_J}\frac{1}{ln\left(\frac{B_x\lambda_J}{H_0\lambda_{ab}}\right)}, \tag{3}$$

and the limit $B_x \gg H_0$ corresponds to strongly overlapping Josephson cores. Note that the amplitude of the zigzag modulation satisfies the condition $u \ll \lambda_{ab}$, and then it is a relatively small deformation of the vortex line consisting of the PVs.

Now let us calculate the interaction of such two zigzag vortex lines at the distance x, both located in the $(\mathbf{x}, \mathbf{z})$ plane, see Fig. 1. Those characteristic distances, which are of interest for us, are much smaller than λ_c, and then in the calculation of the interaction energy we may neglect the Josephson coupling and use the general form of the energy in the pure electromagnetic limit (1). In our case, the London vectors of the layers are $\mathbf{\Phi}_{2n}(\mathbf{k}) = e^{i\mathbf{uk}}\mathbf{\Phi}(\mathbf{k})(1 + e^{-i\mathbf{xk}})$ and $\mathbf{\Phi}_{2n+1}(\mathbf{k}) = e^{-i\mathbf{uk}}\mathbf{\Phi}(\mathbf{k})(1 + e^{-i\mathbf{xk}})$. Performing in (1) the necessary summation and using for G_k its expansion for $ks \ll 1$ (which is perfectly justifiable when the distances of interest are larger than the interlayer distance s), we finally obtain the following expression for the interaction energy per one layer

$$E_{int}(x) = \frac{s\Phi_0^2}{2\pi\lambda_{ab}^2}\int\frac{d^2\overrightarrow{k}}{(2\pi k)^2}\left[\begin{array}{c}\cos(xk_x) - \frac{\cos(xk_x)}{2\left(1+k^2\lambda_{ab}^2\right)} \\ -\frac{\cos[k_x(x-2u)]}{4\left(1+k^2\lambda_{ab}^2\right)} - \frac{\cos[k_x(x+2u)]}{4\left(1+k^2\lambda_{ab}^2\right)}\end{array}\right] \tag{4}$$

Performing firstly the integration over k_y and then over k_x, we may present the interaction energy as

$$E_{int}(x) = s\left(\frac{\Phi_0}{4\pi\lambda_{ab}}\right)^2\left[\begin{array}{c}K_0\left(\frac{x}{\lambda_{ab}}\right) + \frac{1}{2}K_0\left(\frac{x+2u}{\lambda_{ab}}\right) \\ +\frac{1}{2}K_0\left(\frac{x-2u}{\lambda_{ab}}\right) + ln\left(\frac{x^2-(2u)^2}{x^2}\right)\end{array}\right], \tag{5}$$

where K_0 is the modified Bessel function of zero order. At long distances $x \gg \lambda_{ab}$, the Bessel function $K_0\left(\frac{x}{\lambda_{ab}}\right)$ decays exponentially, and the leading contribution comes from the last term in (5), which gives $E_{int}(x) \approx -s\left(\frac{\Phi_0}{2\pi\lambda_{ab}}\right)^2\frac{u^2}{x^2}$, i.e. at long distances the net interaction between the zigzag PVs lines is an attraction! At short distances, the interaction is repulsive, as usual $E_{int}(x) \approx -s\left(\frac{\Phi_0}{2\pi\lambda_{ab}}\right)^2 ln\left(\frac{\lambda_{ab}}{x}\right)$. The overall behavior of the interaction energy is presented in Fig. 2 for different deformation parameters $\varepsilon = \frac{u}{\lambda_{ab}} \sim \frac{H_0}{B_x}\frac{\lambda_{ab}}{\lambda_J}$. The minimum energy may be easily found for small deformation parameter ε using the asymptotic of the Bessel function $K_0(z) \approx \sqrt{\frac{\pi}{2z}}exp(-z)$. With logarithmic accuracy the minimum realizes at

$$x_{min} \approx 2.5\lambda_{ab}ln\left(\frac{3}{\varepsilon}\right) \tag{6}$$

and

$$E_{int}(x_{min}) \approx -s\left(\frac{\Phi_0}{4\pi\lambda_{ab}}\right)^2\frac{\varepsilon^2}{ln\left(\frac{1}{\varepsilon}\right)}. \tag{7}$$

This result leads to the important physical conclusion that in the presence of Josephson vortices, the AVs, due to the long range attractive interaction, will form chains. The equilibrium distance between the vortices in the chain is given, with logarithmic accuracy, by the same expression (6) as in the two vortices case. In fact, the energy of the AV in chain is lower than the energy of a solitary vortex, and with the increase of B_z the vortices will start to penetrate in form of chains. Further increase of B_z will increase the number of chains without changing in first approximation the distance between vortices in chains. Only when the distance between the chains reaches the value of the order of λ_{ab}, the formation of the usual Abrikosov lattice will occur. In the low field regime $B_z \ll H_{c1}^c = \frac{\phi_0}{4\pi\lambda_{ab}^2}\ln\left(\frac{\lambda_{ab}}{\xi}\right)$, the well defined dense vortex chains must be present.

Now let us discuss the influence of the parallel field on the vortex chain state. In the high field limit $B_x \gg H_0$, the increase of the parallel field decreases the deformation parameter, see (3), and thus, following (6), will slightly increase the intervortex distance inside the chain. A more important effect, actually, is the strong decrease of the potential dip in the intervortex interaction energy (7) which will result in the melting of the vortex chain. The vortex chain occurs to be more stable at lower parallel field. However, the above approach is valid for $B_x > H_0$, and then the maximum stability of the chain is expected at $B_x \sim H_0$. At lower parallel field, the distance between Josephson vortices along **z**-axis becomes much larger than $2s$, as well as the distance between deformed parts of the PVs line. Evidently, the attraction between AVs, due to its deformation, will weaken, and such a case needs a special analysis which is presented below.

Let us consider the limit of weak parallel field, when the Josephson vortices are well separated and the distance D between them along **z**-axis strongly exceeds the interlayer distance s, i.e. $B_x \ll H_0$, see Fig. 3. As it has

been demonstrated in [12], if the JV is located between the layers 0 and 1, the pancake displacements u_n on the n^{th} layer in the case of the single AV is

$$u_n \approx \frac{2C_n}{\left(n-\frac{1}{2}\right)}\frac{\lambda_{ab}^2}{\lambda_J}\frac{1}{ln\left(\frac{\lambda_J}{\lambda_{ab}}\right)}, \tag{8}$$

where C_n is a numerical coefficient of order one. Note that as it may be expected, this expression for $n = 1$ is of the same order of magnitude as (3) at the limit of its applicability at $B_x \sim H_0$.

To treat this situation, it is convenient to add a fictitious pair of pancake vortex and antivortex at the central line of the AV, see Fig. 3. Then, the obtained configuration will be equivalent to two straight Abrikosov vortices and to two vortex-antivortex pairs at the distance x in each layer. Vortex and antivortex are separated by a distance u_n, and we are coming to the problem of the interaction of such magnetic dipoles. Firstly, note that the interaction between dipoles in the same layer is attractive, and it may be directly calculated with the help of (1):

$$E_{n,n}^d(x) \approx -\frac{s\Phi_0^2}{8\pi^2\lambda_{ab}^2}\frac{u_n^2}{x^2}. \tag{9}$$

It is much larger than the interaction between dipoles from different layers n and m, which may be attractive or repulsive. From (1) it may be estimated as $\left|E_{n,m}^d(x)\right| \approx \frac{s\Phi_0^2}{8\pi^2\lambda_{ab}^2}\frac{u_n u_m}{x^2}\frac{s}{\lambda_{ab}}$ for $s\left|n-m\right| \ll \lambda_{ab}$, i.e. containing the small additional factor $\frac{s}{\lambda_{ab}} \ll 1$. Moreover, for $s\left|n-m\right| > \lambda_{ab}$, this interaction decays exponentially. Taking into account that u_n varies as $u_n \approx \frac{1}{\left(n-\frac{1}{2}\right)}$ near the JV, we may finally conclude that the main contribution to the dipole interaction energy is coming from the interaction in the same layer, and it may estimated (per period D of the modulation of PVs line along **z** axis) as

$$\begin{aligned} E_{att}(x) &\approx -\frac{s\Phi_0^2}{8\pi^2\lambda_{ab}^2}\frac{1}{x^2}\sum u_n^2 \\ &\approx -\frac{s\Phi_0^2}{8\pi^2\lambda_{ab}^2}\frac{u_1^2}{x^2}2\left(C_1^2+\frac{C_2^2}{3^2}+\frac{C_3^2}{5^2}+...\right) \qquad (10) \\ &\approx -\frac{s\Phi_0^2}{\lambda_{ab}^2}\frac{u_1^2}{x^2}. \qquad (11) \end{aligned}$$

The main contribution to the repulsion energy is coming from the straight AVs interaction, and per period D at distances $x \gg \lambda_{ab}$ is

$$E_{rep}(x) \approx \frac{D\Phi_0^2}{8\pi^2\lambda_{ab}^2}\sqrt{\frac{\pi\lambda_{ab}}{2x}}exp\left(-\frac{x}{\lambda_{ab}}\right). \tag{12}$$

As in the case of the dense JVs lattice, the prevailing interaction between AVs at long distances is an attraction, and it will lead to the vortex chain creation. However, as the attraction is coming only from the PVs near JVs, the relative strength of attraction is much smaller (other PVs contributing only to the repulsion). In the result, the distance between AVs in the chain may be estimated as

$$x_{min} \approx 2.5\lambda_{ab} ln\left(\frac{1}{\widetilde{\varepsilon}}\right), \tag{13}$$

where $\widetilde{\varepsilon} = \frac{u_1}{\lambda_{ab}}\sqrt{\frac{s}{D}} \approx \frac{\lambda_{ab}}{\lambda_J}\left(\frac{B_x}{H_0}\right)^{1/4}$. And the energy gain due to the vortex chain formation per pancake is

$$E_{int}\left(x_{min}\right) \approx -s\left(\frac{\Phi_0}{4\pi\lambda_{ab}}\right)^2\frac{\widetilde{\varepsilon}^2}{ln\left(\frac{1}{\widetilde{\varepsilon}}\right)}, \tag{14}$$

i.e. it is decreasing with the decrease of B_x. Then, in the case $B_x \ll H_0$, the perpendicular vortices also appear as vortex chains, and when the perpendicular field increases it will lead firstly to the increase of the number of these chains, each chain located at JVs. The distance between the chains will be an integer number of the distance between JVs along **y**-axis. Finally, when all JVs will contain chains, with equilibrium distance $x_{\min}$ between AVs, further increase of B_z will lead to the appearance of additional vortices in chains, decreasing the intervortex distance below $x_{\min}$. In such a case, the neighboring vortices repeal each other in the chain, but the repulsion energy is compensated by the gain of energy due to the trapping of the AVs by the JVs. However with further increase of B_z, the repulsion energy will overcome the trapping energy and the formation of the usual Abrikosov lattice will start. Namely this case corresponds to a mixed vortex chain-vortex lattice observed in [5,6]. It is interesting that in low perpendicular field $B_z \ll H_{c1}^c$ and in the limit $B_x < H_0$ the mixed vortex chain-vortex lattice may transform into a purely chain state with the increase of B_x. Indeed, the number of JVs increases, permitting therefore to accommodate all AVs. This type of behavior could be responsible for peculiar results on vortex lattice melting in the presence of JVs obtained in [16].

Comparing the results for low ($B_x \ll H_0$) and high ($B_x \gg H_0$) parallel field limits, we see that the distance between AVs in chain is always around λ_{ab} and slightly varies with B_x. On the other hand, the energy of vortex coupling in a chain is maximal for $B_x \sim H_0$, and then they are more stable at this condition. At low or high B_x limits, we may expect the melting of vortex lines.

Note also that the vortex chain state could also reveal an anisotropy of the critical current, the current along y axis will provoke the motion of the chain as a whole, while the current along x axis will provoke the vortex detachment from the chain.

In conclusion, we predict a new qualitative effect, the long range attraction between AVs in the crossing vortex structure appearing in highly anisotropic layered superconductors. Such an effect is somewhat reminiscent

to the tilted vortex attraction in superconductors with moderate anisotropy [1,2]. If we consider the line between neighboring layers with π phase difference as the center of a JV, the PVs displacement disrupts this line at the distance $2u$. So we may consider the resulting structure as a tilted vortex line made from PVs and JVs parts (see the wavy line in Fig. 3). However the very important difference, with the straight tilted vortices, is that in the highly anisotropic case the vortex attraction is completely controlled by the parallel component of the magnetic field only.

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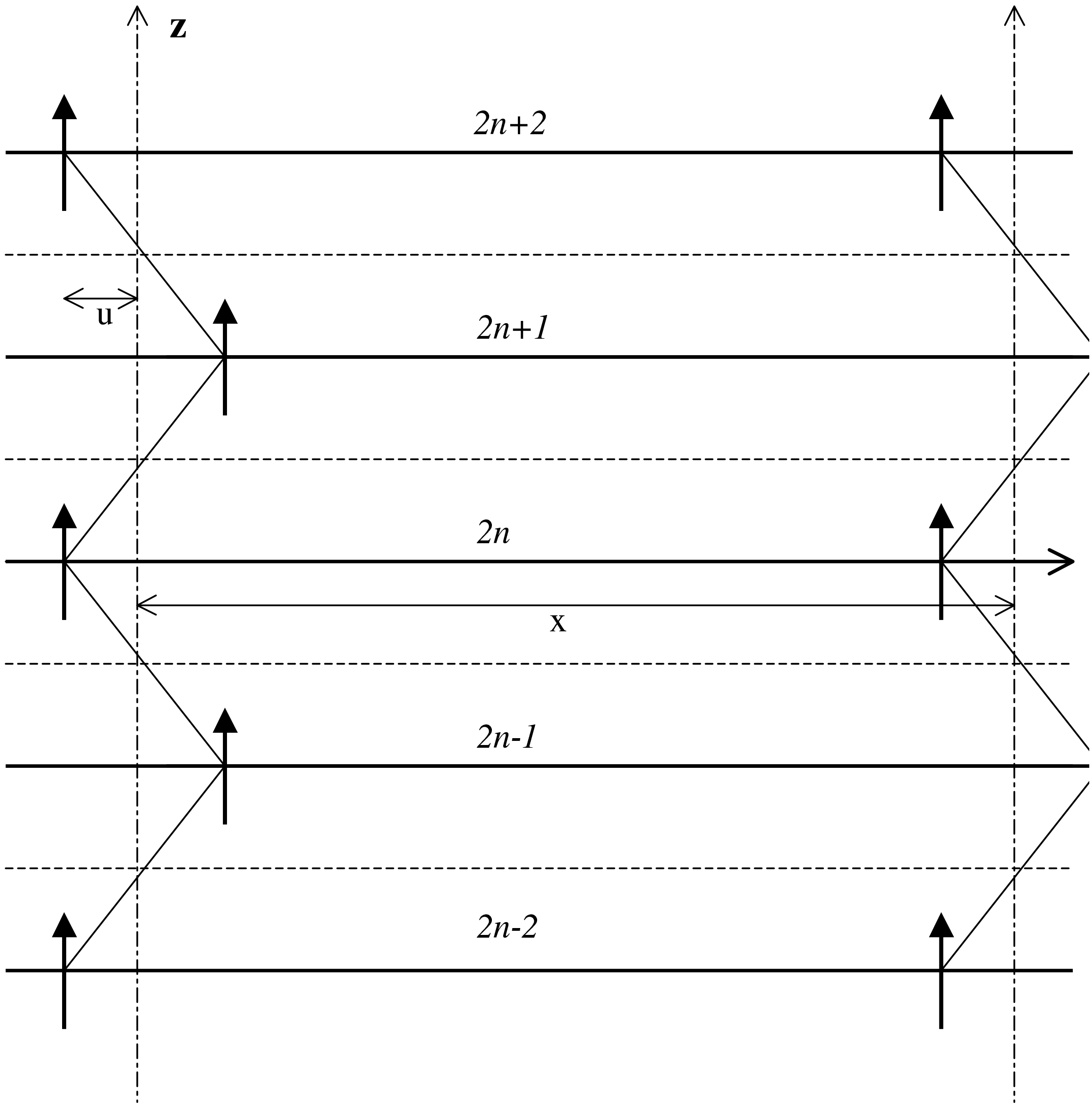
Figure Captions

FIG. 1. Zigzag deformation of the PVs stacks in high parallel magnetic field $(B_x \gg H_0)$, directed along the **x**-axis. u is the amplitude of deformation and the centers of PVs stacks are at the distance x. JVs are presented by dashed lines.

FIG. 2. The energy of interaction between two zigzag vortices as the function of the distance between them for different deformation parameter $\varepsilon = \frac{u}{\lambda_{ab}}$ values.

FIG. 3. Schematic picture of deformed AV lines in case of relatively weak parallel magnetic field $(B_x \ll H_0)$. The distance between the centers of AV lines is x. The JVs are presented by dashed lines. The period D of JV's lattice along the **z**-axis is much larger than the interlayer distance s. The PVs separation breaks the JV line, these parts are presented by cross, and the resulting "tilted vortex line" is depicted by a wavy line. On the upper layer, the fictitious vortex-antivortex pair is shown at the center of the AV. This procedure restores a straight AV line with additional vortex dipole presented as a shaded region.



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u
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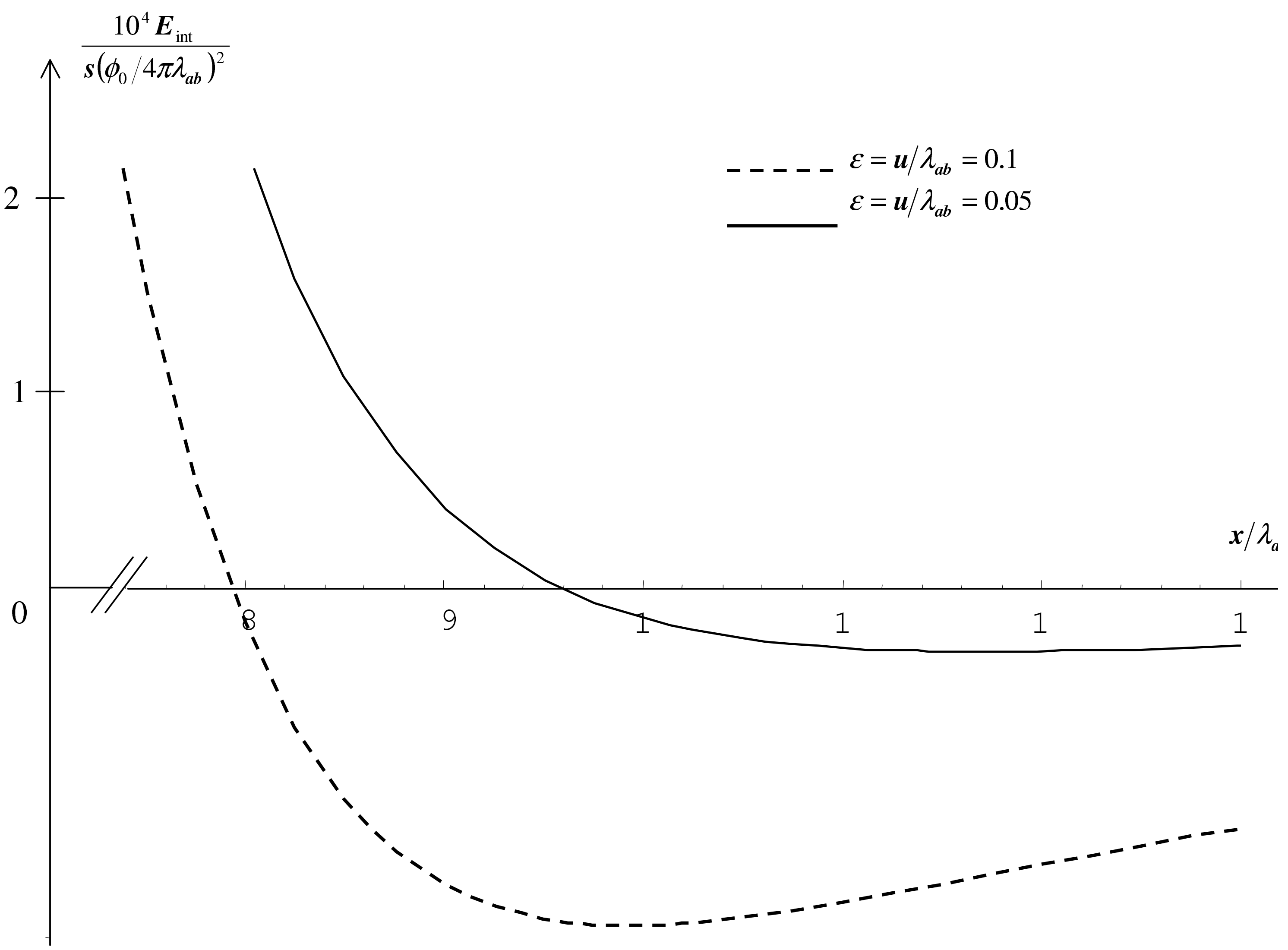
$\frac{10^4 E_{\text{int}}}{s(\phi_0/4\pi\lambda_{ab})^2}$
2
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x/λ_a
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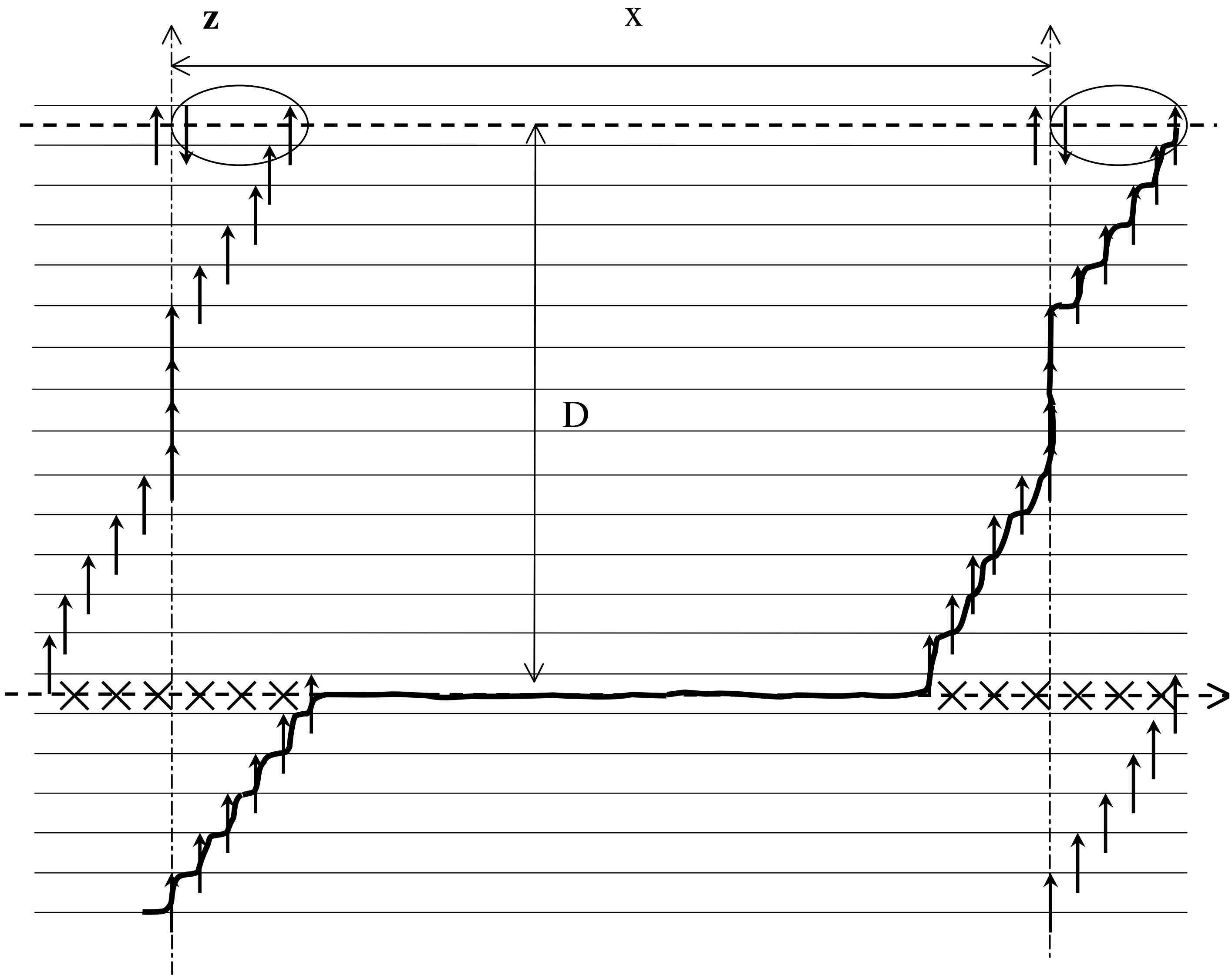
$\varepsilon = u/\lambda_{ab} = 0.1$
$\varepsilon = u/\lambda_{ab} = 0.05$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